§2.9—Derivatives of Exponential Functions

Example 1:

Sketch the graph of $f(x) = e^x$, then, on the same set of axes, sketch a possible graph of $f'(x)$. What do you notice? Confirm by sketching $f'(x)$ using your calculator's NDERIV capability.

Derivative of e^x

$\dfrac{d}{dx}\left[e^x\right] = e^x$. If u is a differentiable function of x, then $\dfrac{d}{dx}\left[e^u\right] = e^u \cdot u'$ (Chain Rule)

Example 2:

Find $\dfrac{dy}{dx}$ if $y = e^{\left(x + x^2\right)}$

Example 3:

Using your calculator, graph $f(x) = 2^x$ and $f'(x)$ using NDERIV. What do you notice? Do the same for $g(x) = 5^x$ and $g'(x)$.

General Derivative of b^x

$\dfrac{d}{dx}\left[b^x\right] = b^x \cdot \ln b$. If u is a differentiable function of x, then $\dfrac{d}{dx}\left[b^u\right] = b^u \cdot \ln b \cdot u'$ (Chain Rule)

Example 4:

At what point on the graph of the function $y = 2^t - 3$ does the tangent line have a slope of 2?

Example 5:

Evaluate the following.

a) $\dfrac{d}{dx}\left[7^{-3/x}\right]$

b) $\dfrac{d}{dx}\left[e^{\ln x}\right]$

c) $\dfrac{d}{dt}\left[\left(e^{-t} + e^t\right)^2\right]$

d) $\dfrac{d}{dx}\left[x^2 \cdot 5^{\sin^2(4x)}\right]$

e) $\dfrac{d}{dx}\left[e^3 x\right]$

Example 6:

Find the coordinates of any points where $f(x) = xe^x$ has a horizontal tangent line.

Example 7:

Find the equation of the tangent line to $y = x^2 e^x - 2xe^x + 2e^x$ at $(1, e)$

Example 8:

Find $\dfrac{dy}{dx}$ for $e^{xy} + x^2 - y^2 = 10$

Example 9:

Find $\dfrac{d^2 y}{dx^2}$ for $y = (3 + 2x)5^{-3x}$

Example 10:

(Calculator Permitted) A glass of cold milk from the refrigerator is left on the counter on a hot summer day.

Its temperature, F, (in degrees Fahrenheit) after sitting on the counter t minutes is $F = 72 - 30(0.98)^t$.

(a) What is the temperature of the refrigerator?

(b) What is the temperature of the room?

(c) How fast is the milk warming when it is first removed from the refrigerator? Explain your answer in a complete sentence.